A self calibration method using a soft clustering procedure for eye movement recordings

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Abstract

A nearly automatic method for calibrating eye movement records has been developed. This very robust method is based on a soft clustering algorithm which allows exploration of the whole range of eye movement records for reliable calibration. In contrast to many other methods which carry out the calibration on several discrete points, this method is suitable for continuous determination of the transfer function of the eye movement transducer. Moreover it simultaneously uses the combined properties of vestibulo-ocular reflex, neck-to-eye reflex and smooth pursuit system to reach approximately a unity gain and zero phase lag (in subjects with no severe vestibular disorders or ocumomotor palsy). In addition, this method does not rely heavily on the degree of attention of the subject. The method is particularly suited for the calibration of non linear or noisy transducers like Electro Oculography (EOG). Calibration is performed within a few seconds. So when necessary in clinical applications it is possible to repeat calibrations frequently. © 1998 Elsevier Science B.V. All rights reserved.

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1. Introduction

Among eye movements, saccades are the most rigidly organized. For a given species there is little variation between individuals and still less in a single individual. In addition to the saccadic system, eye movements are controlled by two main reflex systems: the vestibulo-ocular reflex (VOR) and the optokinetic reflex whose roles are to stabilize the visual scene on the retina, despite movements of the head or body for the former, and of the scene itself for the latter. The last main system is smooth pursuit which allows fixation to be maintained on a slowly moving object. The saccadic system has a frequency spectrum for position ranging approximately from 0 to 40 Hz., whereas the smooth pursuit system and the reflex compensatory systems (vestibulo-ocular and optokinetic) have a fairly constant phase and gain response within a range of 0.01 to 1 or 2Hz. Thus, subtle variations in the characteristics of one of these oculomotor subsystems (for instance the peak velocity of a saccade, the gain and phase of the smooth pursuit system...) may reveal functional disturbances which are clinically undetectable but correspond to various pathologies in neurology or ophthalmology. However, the quantification of such parameters requires reliable and accurate measurements of rotations of the eye-ball within the orbit.

The main qualities required of a transducer are repeatability, linearity of transfer function, and an amplitude range and frequency bandwidth larger than those of the measured variable. Most optical systems, based either on corneal reflection or on iris border detection techniques have quasi-linear transfer functions, but are suitable for only a limited range of eye movements (20–25°). Low cost optical systems based either on corneal reflection (Stark and Sandberg, 1961) or on iris–sclera border detection (Massé, 1971; Findlay,
require some expertise for proper positioning of the sensor. Also since they are very sensitive to slight displacements of the sensor with respect to the head, they require that the head be perfectly immobilized with a bite board, which limits their use for clinical purposes. Electromagnetic systems, such as the search coil (Robinson, 1963; Collejwin et al., 1975), which have all the qualities of a good transducer, are probably too invasive to be used systematically in humans. The electro-oculogram (EOG) is the most commonly used in clinical practice (Kris, 1958; Shackel, 1960; Prablanc et al., 1986), as it is relatively inexpensive and can be set up with no expertise. Its function is based on the existence of an electrical dipole oriented along the optical axis, which diffuse across the tissue and can be recorded through surface electrodes placed near the canthi of the eye. When the EOG is recorded bitemporally between the outer canthi, the signal is that of an equivalent cyclopean eye, whereas when the electrodes are placed at the inner and outer canthi of one eye, the orbital position of that eye is recorded.

There are several ways to build up a transducer’s transfer function. One is to carry out a calibration at several discrete points and, with apriori knowledge of the shape of the transfer function, to compute a best fitting polynomial using a least squares method. This is the most commonly used method (for transducers known to be fairly linear the degree of the polynomial can sometimes be decreased to one). Although this method is relatively safe for a linear transducer, it depends heavily on the subject’s degree of attention, since he has to keep his gaze anchored on the calibration target. This may be a problem with patients unable to maintain sustained eccentric fixation. A large number of calibration points is necessary in order to eliminate errors. Thus the duration of the calibration makes the method unsuitable for most patients with a diminished level of attention. With non linear eye movement transducers the number of calibration points necessary increases, making the method still more time consuming.

Another means of calibrating an eye movement transducer is to elicit foveal tracking of a target moving sinusoidally from the extreme right and left positions, and thus to explore all points of the eye movement transfer function. As well as requiring great attention on the part of the subject, this method necessitates some hypotheses about the behavior of the smooth pursuit oculomotor system, in particular that it has unity gain and zero phase lag, over the entire range of velocity, i.e. that the image remains on the fovea during the entire calibration phase. A large number of studies suggest that the smooth pursuit system is non linear and learning-dependent and that it therefore has a phase lag which can vary predictably for the same sinusoidal stimulus (Bahill and McDonald, 1981). Some authors have suggested that ‘smooth’ eye movements may in fact appear at the microstructure level as a succession of ramps of given velocity following an internal model velocity (Bahill and McDonald, 1983) (for a review see Carpenter, 1988). Without prediction, the temporal lag of the eye is about 100 ms. In order to reduce the phase lag of smooth pursuit to near zero, the frequency of the stimulus must be reduced as far as 0.1 Hz. At such a low frequency, the time spent in calibration becomes non negligible if the calibration has to rely on more than one cycle, and there is a high probability of loss of attention, even in normal subjects, with the occurrence of saccades bringing the fovea off the target.

The method we describe here makes use of the properties of the vestibulo-ocular reflex (VOR). It is a very fast open loop process with delays of about 10 ms (for a review see Carpenter, 1988) within a range of 1–10 Hz. It has a velocity transfer function with a gain close to 0.7–0.8 and zero phase lag. When the proprioceptors of the neck are stimulated at the same time as the vestibular organs, the total (VOR + Cervico-Ocular Reflex, COR) gain reaches absolute values close to one even in complete darkness (Péisson et al., 1988). By presenting a small luminous fixation point in the body’s sagittal plane, the combined behavior of the subsystems: VOR, COR and smooth pursuit, allows a perfect fixation and gives a global transfer function near unity gain and zero phase lag in the frequency range: 0.1–0.5 Hz (the amplitude of movement is ±40°). Thus for a transducer without noise and for a perfectly attentive subject the trans-

\[ x = t \sin(a) + \varepsilon \]  
\[ y = t \cos(a) - \varepsilon \]

Fig. 1. (a) Illustration of the geometrical basis for calculating eye movement parameters. Head rotation angle \( \alpha \) and eye rotation angle \( \beta \) are measured in the clockwise direction. \( P \) is the fixation point, \( A \) the left eye and \( B \) the right eye. \( O \) represents the centre of the head, \( OU \) its direction. \( AV \) is parallel to \( OU \). The distance from the interocular axis \( AB \) to the fronto parallel line through \( P \) is denoted by \( x \). \( d \) is the distance between the head centre and the interocular axis. \( e \) is half the interocular distance. \( y \) is the distance from \( P \) to \( AV \). When the head moves in the clockwise direction, the eye moves in the counterclockwise direction to stay anchored on the fixation point. For a sufficiently large distance between the head and the fixation point, we have: \( b \approx -a \). Finally \( l \) is the distance between \( O \) and \( P \). With reference to these measures, we have: (1) \( y = l \sin(a) + \varepsilon \) and (2) \( x = l \cos(a) - \varepsilon \). In (1) the + sign is for right eye and – sign for left eye. From these formulae we obtain:

\[ y/x = tg(\beta) = (l \cdot \sin(a) + \varepsilon)/(l \cdot \cos(a) - \varepsilon) \]

(b) The upper graph shows eye and head position as the subject keeps his gaze on the central fixation point and rotates his head bilaterally. In the lower graph the EOG measuring instantaneous eye position is shown. It can be seen how noise, eye blink and drift are added to the signal. More importantly, fixation instability adds offset to the eye position curve. The duration of this offset period is not negligible.
Fig. 1.
ducer transfer function may be obtained from the ratio of eye movement to head movement, supposing the fixation target to be at infinity (for a near fixation target, the real eye movement is given, to a good approximation, by a simple formula linking it to the head movement. Thus:
\[
b = \arctan(\frac{\sin(a)}{\cos(a) - d/l})
\]
for a binocular recording, corresponding to the equivalent cyclopean eye. For a monocular recording, the following formula is used (+ for the left eye and − for the right eye)
\[
b = \arctan((l \sin(a) + e)/l(\cos(a) - d))
\]
where \(a\) and \(b\) are instantaneous angles of head and eye respectively. For an illustration of these parameters (see Fig. 1a).

\(l\), is the distance from the head’s center of rotation to the target, \(2e\), is the interocular distance, and \(d\) is the distance between the center of rotation of the head and the line joining the two eyes (see Fig. 1a).

This method can be used with any type of d.c. eye movement transducer, but the difficulty of eliminating noise will depend on the technique. We will consider the EOG which is the noisiest, but is also the easiest to use in clinics: its equivalent noise may extend from \(±0.5\) to \(1°\) within the physical bandwidth of the eye movements. In addition, when using the EOG technique, a very low frequency drift of the electrical dipole amplitude may occur; this results in a change in the gain of the EOG. Recalibration is therefore periodically necessary. Moreover, the noise has a high and medium frequency component from the electromyographic activity of the facial muscles (extending mainly from 1000 Hz to as low as 30 Hz, i.e. within the power spectrum of saccades). Despite improvements in the quality of disposable electrodes with low polarization and low impedance, a very low frequency drift still occurs, even after a stabilization period. This drift is due partly to the skin to electrode impedance change, whose spectrum extends from 0 to 0.05 Hz, i.e. much lower than the frequency of slowly alternating head movements (0.1–0.5 Hz).

In addition subjects frequently have eyelid blinks which result, with most recording techniques (except the electromagnetic technique, which is insensitive to physical obstacles to the light), in a low frequency pulse artifact. Moreover, especially with patients, instability of fixation may produce sudden saccades, bringing the eye off the foveal target fixation for a few hundred milliseconds, although the slow phase velocity continues correctly (see Fig. 1b); finally there is a saccade bringing the eye back to foveal target fixation.

Classical filtering techniques using the direct and inverse Fourier Transform fail to remove these artifacts. In order to eliminate eye movement artifacts as well as fixation instability and noise, we have used a method originally developed by Arzi and Magnin (1989) for pattern recognition in nystagmic eye movements. This method is based on fuzzy set theory and allows eye movements to be separated into slow- and fast-phases.

2. Methods

2.1. Principle

The basic idea is to find out which part of the eye movement recording is actually part of the calibration and which is either an artifact or an ‘off’ section, and then to replace those erroneous sections by extrapolations from the accurate sections.

The problem can be formulated as follows: parts of a given signal are mixed with various artifacts during which the signal is missing. Some information about the approximate shape of the signal is available but almost nothing is known about the waveform of the artifacts. In our case, the signal is eye movement in response to a known stimulus; artifacts are spontaneous saccades or other eye movements such as blinking. The problem consists of eliminating the artifacts and reconstructing the signal. Not only are some parts of the signal (during artifacts) missing, but artifacts add offsets to the available parts of the signal. To reconstruct the latter by an algorithm, it is necessary first to recognize the available parts of the signal from the artifacts, then to add appropriate offsets to successive pieces of the signal in order to interpolate between them. The main difficulty is to recognize the pieces of signal from the artifacts. Note that this problem is different from that of extraction of a signal from noise, in which it is supposed that the signal exists everywhere but it is linearly superposed on noise.

To solve this problem, we suppose that a pattern whose approximate shape is known, is distinguishable in the signal (or in one of its derivatives). All data points are clustered into two classes. Data points belonging to parts of the eye movement curve corresponding to eye response to the given stimulus are assigned to the first class. The second class comprises all other points irrelevant to eye response to the known stimulus. It is clear that any classification algorithm based on strict criteria fails to perform reliably in this situation, because stimulus related parts of the data can be defined only approximately. Moreover, any criterion for the distinction between the two classes can only be defined vaguely. For these reasons, the classification is done softly in the sense that the classes are fuzzy subsets of all the data points. It is important to mention here that the method described below is different from fuzzy clustering algorithms developed earlier (see for example Windham, 1983).
2.2. Classification algorithm

The classification is performed by an iterative algorithm. The two complementary sets (classes) are defined by a membership function as follows: To each point \( p \), whose coordinates are \( x \) and \( y \), is associated the value of the membership function \( \mu_f(p) \). This value defines the degree to which the shape of the segment of the curve including points in a certain neighbourhood around the point \( p \) is similar to the global shape of the stimulus. The mathematical formula defining the stimulus is \( y = f(x) \) (the subscript \( f \), in \( \mu_f(p) \), refers to the same \( f \) as in \( y = f(x) \)). This formula can be for example a sinusoidal function or a ramp according to a priori knowledge about the stimulus. In the absence of any information about the shape of the signal, a polynomial of sufficiently high degree can be chosen. Examples of a polynomial function are given near the end of the paper.

All other points are clustered in the complement of the first class (in the sense of fuzzy-set theory that means that the complementary class is defined by the membership function: \( 1 - \mu_f(p) \)). \( \mu_f(p) \) can also be seen as the degree of coherence of the piece of signal in the neighbourhood of \( p \) (and including \( p \)), with the function \( f \). For this reason we will call this membership function ‘the coherence function’.

The assignment of points to a class or to its complement is what we call discrimination into crisp (non fuzzy) subsets. This is equivalent to replacing the membership function of a fuzzy set by the ‘characteristic function’ of a crisp set. The latter function can take only 1 or 0 as its values, depending on whether or not a point belongs to the set. Discrimination is done by choosing some: ‘level set’ \( f_a \). A level set (also called ‘a-cut’) is defined to be the crisp subset of all points whose grade of membership is equal to or greater than a fixed threshold (or level) \( a \).

In general, defining a membership function, based on some imprecise criterion (for example the criterion of difference of velocity between slow- and fast-phase of eye movements, or the criterion of similarity of a recorded signal to a shape whose approximate mathematical formula is known), results in a function whose transition between total membership (\( \mu_f(p) = 1 \)) and non membership (\( \mu_f(p) = 0 \)) is slow.

One important consequence of slow transition of the membership function from high to low values (or vice versa), is that many points will have membership values far from zero and one (see Fig. 2). This will be a source of difficulty in the final decision to assign these points to either class. Moreover, a slight variation (\( \Delta V \) in the choice of the level, results in a large variation (\( \Delta V \)) in the classification of points. Moreover data points whose membership grade satisfy the relations: \( 1 - a < \mu_f(p) < a \) can not be assigned to either class. The number of these points, for fuzzy subset defined by a membership function with gradual transition between non membership and total membership, may be rather large. These facts are illustrated in Fig. 2.

![Fig. 2. Illustration of the discrimination power of a membership function.](image-url)
So, we will define the discrimination power of \( \mu_f(p) \) in such a way that it takes high values for membership functions that have a sharp transition between 0 and 1 and low values for membership functions with slow, gradual transition between these values.

Beginning by a membership function with a poor discrimination power, our algorithm recalculates, in an iterative process, this function and in each iteration the discrimination power is improved until it satisfies some criterions defined appropriately for final decision. Our purpose is to obtain a membership function with an abrupt transition between total membership and non membership. This allows a discrimination which is almost independent of any level chosen and the undetermined subset, described above, is reduced almost to the null set. So, to explain the structure of the fuzzy clustering algorithm, it is necessary to introduce the following concepts:

1. **Discrimination power** \( D \), is a measure of the separation of the fuzzy set, defined by \( \mu_f(p) \), from its complement defined by \( 1 - \mu_f(p) \). For our purposes, it is appropriate to define \( D \) as follows:

\[
D = \sum |\mu_f(p) - (1 - \mu_f(p))| = \sum |2\mu_f(p) - 1|
\]

The classification algorithm attempts to maximize \( D \).

2. **Global pattern**: \( y = f(x) \), is a mathematical formulation of the global shape of the stimulus. The approximate shape of this function is known from a priori information concerning the stimulus but its exact formula is obtained from the data as explained below.

3. **Coherence function** \( \mu_y(p) \). This function defines, for each point \( p \), whose coordinates are \( x \) and \( y \), the local similarity (or coherence) of the shape of the set of points around \( p \), to the corresponding part of the global pattern \( y = f(x) \). The construction of this function is the most important part of the algorithm. A short description of this procedure is given in the appendix, but for more details see Arzi and Magnin 1989.

We may now summarize our clustering algorithm. It is based on an iterative procedure as follows:

1. **Initialize** \( \mu_f(p) = 1 \) for all points of the compound curve. This means that we begin with a flat membership function which is refined during successive iterations.

2. **Repeat until convergence or a maximum number of iterations**:
   2.1. Calculate parameters of the curve representing the global pattern by minimizing the following sum: \( S = \sum \mu_f(p)(y_i - f(x_i))^2 \) where \( x_i \) and \( y_i \) are the coordinates of point \( p_i \) of the compound curve.

2.2. Calculate the coherence (membership) function \( \mu_y(p) \) based on \( y = f(x) \) and its discrimination power. This is done as described in the appendix.

At each iteration, the transition between total membership and non membership of the coherence function becomes sharper and consequently the membership function obtains higher discrimination power.

The criterion to stop the iteration (convergence) is simply when the improvement of the coherence function becomes stationary, meaning that its discrimination power has attained its optimal value. The convergence is usually attained after two or three iterations; occasionally seven are required.

### 3. Experiment and procedure

#### 3.1. Eye movement recording

Horizontal eye movements were recorded by the electro-oculographic method (EOG). Recordings were performed either from the seeing eye with monocular viewing or binocularly with outer canthi electrodes; in the latter case the recorded signal is the equivalent of the theoretical cyclopean eye. The fixation target was a small i.e.d. 1.5 m from the subject in his sagittal plane, at the height of the interocular horizontal plane.

Eye movement was recorded through a D.C. isolated differential amplifier with computer controlled gain and an offset of 16 bits, (allowing almost the full range of the converter to be used). The raw signal was filtered by an analog low-pass 40 Hz cut off filter (Butterworth –12 dB/oct.) followed by sampling with an 8 bit A/D converter at 100 Hz frequency. The undersampling and the low resolution converter were chosen in order to demonstrate the efficiency of the fuzzy method.

Head movements were monitored through a potentiometer device mounted on a helmet. Two types of head movements were used: (i) free active horizontal head movements with a light helmet; (ii) passive horizontal head movements, which were produced by a servo-motor moving the head sinusoidally in the horizontal plane.

This was achieved through an helmet on which was mounted a bite board. By using a laser mounted on the helmet during the preliminary tests, the ‘dead’ zone was found to be less than 0.5°, when the subject tried to counteract an imposed static position of the servomotor; in the passive condition the head was sinusoidally moved at about 0.2 Hz for 50 s, with an amplitude of ±40°, the subject being instructed to keep his neck relaxed while maintaining fixation on the central point.

The same A/D channel as for eye movement recording was used for head movement recording with the same analog filter and sampling frequency.
Fig. 3. Behavior of the clustering algorithm in a simple case. The upper graph shows the membership function at the end of the optimization procedure. Note the membership function’s rapid transition from non membership to total membership. The membership grade drops to 0 whenever velocity lies outside the dotted strip in the lower graph. The width of this strip is defined in the text. On the lower graph the global pattern of eye velocity (after optimization) is superimposed on the eye velocity curve and is illustrated by the heavy curve. This particular curve could be obtained by an ordinary best fitting curve. In other examples, ordinary best fitting procedures cannot find the global pattern of the velocity curve. Fig. 3b. Transfer function relating actual cyclopean eye position signal to the reconstructed EOG voltage is illustrated. See text for details.

4. Results

The behavior of the algorithm is illustrated on Fig. 3a, where the calibration of a normal subject was performed binocularly while the head was passively moved at 0.2 Hz with an amplitude of \( \pm 40^\circ \), (i.e. peak velocity 50°/s). At such a velocity the cyclopean eye position signal is exactly opposite to the head position and it keeps the fixation point in perfect foveal vision. Eye velocity was estimated by differentiation of the position signal using a 14-coefficient digital FIR filter with cut-off at 5.0 Hz. The estimate of the slow-phase eye movement is represented in heavy lines. The dotted strip represents the velocity interval within which the eye movement is likely to be a slow phase movement, it corresponds to a high value of the coherence function. It can be seen clearly in the upper graph that the membership function of the slow phase velocity drops down when the raw velocity crosses the dotted strip. The lower graph is the transfer function relating the actual cyclopean eye position signal to the reconstructed EOG voltage. The advantage of this method compared to
Fig. 4. Another example of the behavior of the clustering algorithm. The signal contains random off saccades. It can be seen on the upper graph that the value of the membership function is zero during certain intervals. These intervals correspond to random off saccades made by the subject. An ordinary best fitting curve does not yield these results. As can be seen on the lower graph, the heavy curve which represents the global pattern of eye velocity after optimization of the membership function, perfectly estimates eye velocity by removing noise, off saccades and other non relevant parts of the signal. Fig. 4b. Construction of the cumulative slow phase eye position curve for the same signal as the one whose velocity curve is shown in Fig. 4a. The detected slow phases are represented by the heavy curve and indicated by arrows. In spite of long off saccades and eye blinks, the detection of slow phases is reliable.

more classical filtering methods is not apparent in this example since simple FIR filtering of the signal at 1 or 2 Hz would have given excellent results.

The power of the algorithm appears more clearly in Fig. 4a when the same subject, under exactly the same conditions of passive head rotation as above, was instructed to keep his gaze on the central fixation point and to make random off saccades in various directions, thus simulating pathological eye movements with large fixation instabilities. The smooth pursuit velocity estimate follows very closely what an expert investigator would report from inspecting the eye position, note particularly the blink towards the end of the record.

The behavior of the algorithm was also tested with non-sinusoidal natural head movements, and Fig. 5 represents a binocular (cyclopean) selfcalibration performed with free-head movement where the subject was instructed to move his head back and forth while keeping his gaze stationary on the fixation target. The velocity estimate fits the envelope of the slow phase eye movement. No particular hypothesis is required to determine the mathematical formula represented by this enveloppe. It is sufficient to belong to a family of simple mathematical functions which can fit approxi-
Fig. 5. The behavior of the algorithm for a non sinusoidal signal. The curves correspond to those in Fig. 4. In this case, a polynomial of fifth degree was used to fit the global pattern of eye velocity.

5. Conclusion

The technique of calibration of eye movements by the head itself is an easy way to perform the most critical phase for reliable eye movements recordings, but it has never been feasible for clinical purpose because of the frequent artifacts present in patients’ recordings. The originality of the present method lies in the robustness of the automatic algorithm based on fuzzy set clustering. In fact little attention from the...
subject is required in this self calibration technique compared with other techniques such as the smooth pursuit of a sinusoidally moving target. Not only is greater attention required in the latter case, but process is limited by the peak velocity beyond which the smooth pursuit system no longer allows the eye to be locked on the stimulus which is about 25°/s, i.e. half the velocity used in the present experiment. The facility of the present calibration is due primarily to the combined action of several subsystems: the smooth pursuit system, the collicular-ocular reflex, the vestibulo-ocular reflex, which together tend to keep the gaze stationary in space, and the optokinetic system, which tends to stabilize the retinal slip of the whole visual field. Except for patients with severe vestibular lesions, this method could be used for calibration in clinical practice. There has always been a problem when trying to extract quantitative data from the oculomotor system. This is especially true when using the EOG technique, the only cheap and non invasive method available for movements larger than 20° of eccentricity.

Our results show that it may be possible to reconstruct smooth eye movement position from a signal as noisy as the EOG, when additional artifacts, such as blinks, are present, or when a loss of attention produces saccades driving the eye off the fixation target for a few hundreds of milliseconds. This relatively robust method necessitates little intervention of the operator on the signal. For a single cycle of self-calibration the computational procedure using the fuzzy set algorithm and the display of the transfer function takes less than 1 s on a PC 486 running at 66 MHz. This fast procedure allows frequent recalibrations during a recording session, and thus safer recordings.

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Appendix A. Construction of the coherence function

To understand how the coherence function is defined, it is necessary to imagine the \(x\)-\(y\) plane divided into curvilinear strips having the shape of the global pattern \(f(x)\), translated along the \(y\)-axis and each having a certain width.

Each strip is indexed by a number \(k\) and includes the following set of points \(p_i (x_i, y_i)\) (\(x\) represents the time):

\[
\text{str}_k = \{p_i (x_i, y_i)|y_i = f(x_i) + k \cdot \Delta y \pm (\Delta y/2)\};
\]

\(M1 \leq k \leq M2\)

where \(k\) is the strip number. \(M1\) and \(M2\) satisfy the following relations:

\[
M1 \cdot \Delta y = \min_i (y_i - f(x_i))
\]

\[
M2 \cdot \Delta y = \max_i (y_i - f(x_i))
\]

where \(\Delta y\) represents the strip width. It is determined by the following relation:

\[
\Delta y = (\max_i (y_i) - \min_i (y_i))/N
\]

The number \(N\) (which is inversely proportional to \(\Delta y\)) determines the resolution of the algorithm to detect small local variations in the eye velocity curve. Our experiments show that a value between 20 and 30 for \(N\) is suitable but in general it depends on tolerance one can accept in local variations of eye velocity.

The coherence of a piece of eye movement curve is determined by the degree to which it fits into a corresponding part of a single strip. The membership of each point to eye response curve is then determined by the number of points in its neighbourhood which fall in the same strip. This is where local information (neighboring points) and global information (mathematical shape of strips) interact.

For each point \(p_i\), three neighborhoods: \(W1, W2\) and \(W3\) are fixed as follows. Two neighborhoods \(W1\) and \(W2\) are defined by two overlapping temporal windows of width \(w\) and containing \(p_i\). The intersection of \(W1\) and \(W2\) defines \(W3\) and its width is \(w/d\). The latter window which contains \(p_i\) constitutes the third neighbourhood. Then we assign a weight to each strip by counting the number of data points falling in it. This is done for each window. We obtain three distributions for \(\text{str}_k\): \(m1(k), m2(k), m3(k)\).

To each point \(p_i\), which is in the strip number \(k\), and lies in \(W3\) the following membership grade is assigned:

\[
\mu_i (p_i) = m1 \cdot m2 \cdot m3
\]

Parameters \(w, d, \text{and} \Delta y\) (or \(N\) in Eq. (4)) are determined dynamically according to a priori information about the stimulus. For periodic signals, the program assigns a value to \(w\) which corresponds to an appropriate fraction of the period, for example 1/4 of a period. \(\Delta y\) is equal to 1/10 of the range of the stimulus, and \(d\) is initialized to three and then incremented during iterations up to \(w\), so that the intersection of the two windows is initialized to \(w/3\) and at the end reduces to a single point.

After calculation of the membership grade of all points, this value is normalized to obtain a membership grade between 0 and 1 by dividing the membership of all points by their maximum.

References


